TExES | Texas Examinations of Educator Standards

Preparation Manual

135 Mathematics 8–12

Chapter 1

Introduction to the Mathematics 8–12 Test and Suggestions for Using this Test Preparation Manual

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OVERVIEW

The State Board for Educator Certification (SBEC) has approved Texas educator standards that delineate what the beginning educator should know and be able to do. These standards, which are based on the state-required curriculum for students — the Texas Essential Knowledge and Skills (TEKS) — form the basis for the Texas Examinations of Educator Standards® (TExES®) program. This initiative, directed by Texas Education Agency (TEA), will affect all areas of Texas education — from the more than 100 approved Texas Educator Preparation Programs (EPPs) to the more than 7,000 Texas school campuses. This standards-based system reflects TEA's commitment to help align Texas education from kindergarten through college. TEA's role in this K–16 initiative will ensure that newly certified Texas educators have the essential knowledge and skills to teach the TEKS to the state's public school students.

This manual is designed to help examinees prepare for the TExES test in this field. Its purpose is to familiarize examinees with the competencies to be tested, test question formats and pertinent study resources. EPP staff may also find this information useful as they help examinees prepare for careers as Texas educators.

KEY FEATURES OF THE MANUAL

- *List of competencies that will be tested*
- *Strategies for answering multiple-choice questions*
- *Sample test questions and answer key*

If you have any questions after reading this preparation manual or you would like additional information about the TExES tests or the educator standards, please visit the SBEC website at **www.sbec.state.tx.us**.

USING THE TEST FRAMEWORK

The Texas Examinations of Educator Standards (TExES) tests measure the content knowledge required of an entry-level educator in a particular field in Texas public schools. This manual is designed to guide your preparation by helping you become familiar with the material to be covered on the test you are planning to take, identify areas where you feel you may be weak and increase your knowledge in those areas by helping you design a study plan.

When preparing for this test, you should focus on the competencies and descriptive statements, which delineate the content that is eligible for testing. A portion of the content is represented in the sample questions that are included in this manual. These test questions represent only a sampling of questions. Thus, your test preparation should focus on the competencies and descriptive statements and not simply on the sample questions.

ORGANIZATION OF THE TExES TEST FRAMEWORK

The test framework is based on the educator standards for this field.

The content covered by this test is organized into broad areas of content called domains. Each domain covers one or more of the educator standards for this field. Within each domain, the content is further defined by a set of competencies. Each competency is composed of two major parts:

1. the **competency statement**, which broadly defines what an entry-level educator in this field in Texas public schools should know and be able to do, and

2. the **descriptive statements**, which describe in greater detail the knowledge and skills eligible for testing.

The educator standards being assessed within each domain are listed for reference at the beginning of the test framework, which begins on page 12. These are followed by a complete set of the framework's competencies and descriptive statements.

An example of a competency and its accompanying descriptive statements is provided below.

SAMPLE COMPETENCY

Mathematics 8–12

COMPETENCY 001
THE TEACHER UNDERSTANDS THE REAL NUMBER SYSTEM AND ITS STRUCTURE, OPERATIONS, ALGORITHMS AND REPRESENTATIONS.

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SAMPLE DESCRIPTIVE STATEMENTS

The beginning teacher:

A. Understands the concepts of place value, number base and decimal representations of real numbers.

B. Understands the algebraic structure and properties of the real number system and its subsets (e.g., real numbers as a field, integers as an additive group).

C. Describes and analyzes properties of subsets of the real numbers (e.g., closure, identities).

D. Selects and uses appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.

E. Uses a variety of models (e.g., geometric, symbolic) to represent operations, algorithms and real numbers.

F. Uses real numbers to model and solve a variety of problems.

G. Uses deductive reasoning to simplify and justify algebraic processes.

H. Demonstrates how some problems that have no solution in the integer or rational number systems have solutions in the real number system.

STUDYING FOR THE TExES TEST

The following steps may be helpful in preparing for the TExES test.

1. Identify the information the test will cover by reading through the test competencies (see Chapter 3). Within each domain of this TExES test, each competency will receive approximately equal coverage.

2. Read each competency with its descriptive statements in order to get a more specific idea of the knowledge you will be required to demonstrate on the test. You may wish to use this review of the competencies to set priorities for your study time.

3. Review the "Preparation Resources" section of this guide (Appendix B) for possible resources to consult. Also, compile key materials from your preparation coursework that are aligned with the competencies.

4. Study this manual for approaches to taking the TExES test.

5. When using resources, concentrate on the key skills and important abilities that are discussed in the competencies and descriptive statements.

6. Use the study plan document (Appendix A of this guide) to help you plan your study.

NOTE: This preparation manual is the only TExES test study material endorsed by Texas Education Agency (TEA) for this field. Other preparation materials may not accurately reflect the content of the test or the policies and procedures of the TExES program.

Chapter 2

Background Information on the TExES Testing Program

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THE TExES TESTS FOR TEXAS TEACHERS

As required by the Texas Education Code §21.048, successful performance on educator certification examinations is required for the issuance of a Texas educator certificate. Each TExES test is a criterion-referenced examination designed to measure the knowledge and skills delineated in the corresponding TExES test framework. Each test framework is based on standards that were developed by Texas educators and other education stakeholders.

Each TExES test is designed to measure the requisite knowledge and skills that an entry-level educator in this field in Texas public schools must possess. The tests include both individual (stand-alone) test questions and questions that are arranged in clustered sets based on real-world situations faced by educators.

DEVELOPMENT OF THE NEW TExES TESTS

Committees of Texas educators and members of the community guide the development of the new TExES tests by participating in each stage of the test development process. These working committees are composed of Texas educators from public and charter schools, university and EPP faculty, education service center staff, representatives from professional educator organizations, content experts and members of the business community. The committees are balanced in terms of position, affiliation, years of experience, ethnicity, gender and geographical location. The committee membership is rotated during the development process so that numerous Texas stakeholders may be actively involved. The steps in the process to develop the TExES tests are described below.

1. **Develop Standards.** Committees are established to recommend what the beginning educator should know and be able to do. Using the Texas Essential Knowledge and Skills (TEKS) as the focal point, draft standards are prepared to define the knowledge and skills required of the beginning educator.

2. **Review Standards.** Committees review and revise the draft standards. The revised draft standards are then placed on the State Board for Educator Certification (SBEC) website for public review and comment. These comments are used to prepare a final draft of the standards that will be presented to the SBEC Board for discussion, the State Board of Education (SBOE) for review and comment and the SBEC Board for approval. Standards not based specifically on the TEKS, such as those for librarians and counselors, are proposed as a rule by the SBEC Board; sent to the SBOE for its 90-day review; and, if not rejected by the SBOE, adopted by the SBEC Board.

3. **Develop Test Frameworks.** Committees review and revise draft test frameworks that are based on the standards. These frameworks outline the specific competencies to be measured on the new TExES tests. Draft frameworks are not finalized until after the standards are approved and the job analysis/content validation survey (see #4) is complete.

4. **Conduct Job Analysis/Content Validation Surveys.** A representative sample of Texas educators who practice in or prepare individuals for each of the fields for which an educator certificate has been proposed are surveyed to determine the relative job importance of each competency outlined in the test framework for that content area. Frameworks are revised as needed following an analysis of the survey responses.

5. **Develop and Review New Test Questions.** The test contractor develops draft questions that are designed to measure the competencies described in the test framework. Committees review the newly developed test questions that have been written to reflect the competencies in the new test frameworks. Committee members scrutinize the draft questions for appropriateness of content and difficulty; clarity; match to the competencies; and potential ethnic, gender and regional bias.

6. **Conduct Pilot Test of New Test Questions.** All of the newly developed test questions that have been deemed acceptable by the question review committees are then administered to an appropriate sample of candidates for certification.

7. **Review Pilot Test Data.** Pilot test results are reviewed to ensure that the test questions are valid, reliable and free from bias.

8. **Administer TExES Tests.** New TExES tests are constructed to reflect the competencies, and the tests are administered to candidates for certification.

9. **Set Passing Standard.** A Standard Setting Committee convenes to review performance data from the initial administration of each new TExES test and to recommend a final passing standard for that test. The SBEC Board considers this recommendation as it establishes a passing score on the test.

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TAKING THE TExES TEST AND RECEIVING SCORES

Please refer to the current TExES *Registration Bulletin* or the ETS TExES website at **www.texes.ets.org** for information on test dates, test centers, fees, registration procedures and program policies.

Your score report will be available to you in your testing account on the ETS TExES online registration system by 5 p.m. Central time on the score reporting date indicated in the *Registration Bulletin*. The report will indicate whether you have passed the test and will include:

- A total test scaled score. Scaled scores are reported to allow for the comparison of scores on the same content-area test taken on different test administration dates. The total scaled score is not the percentage of questions answered correctly and is not determined by averaging the number of questions answered correctly in each domain.

  – For all TExES tests, the score scale is 100–300 with a scaled score of 240 as the minimum passing score. This score represents the minimum level of competency required to be an entry-level educator in this field in Texas public schools.

- Your performance in the major content domains of the test and in the specific content competencies of the test.

  – This information may be useful in identifying strengths and weaknesses in your content preparation, and can be used for further study or for preparing to retake the test. However, it is important to use caution when interpreting scores reported by domain and competency, as these scores are typically based on a smaller number of items than the total score and therefore may not be as reliable as the total score.

- A link to information to help you understand the score scale and interpret your results.

A score report will not be available to you if you are absent or choose to cancel your score.

For more information about scores or to access scores online, go to **www.texes.ets.org**.

EDUCATOR STANDARDS

Complete, approved educator standards are posted on the SBEC website at **www.sbec.state.tx.us**.

Chapter 3

Study Topics

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TEST FRAMEWORK FOR FIELD 135: MATHEMATICS 8–12

THE DOMAINS

Domain VI
10%

Domain I
14%

Domain V
10%

Domain IV
14%

Domain II
33%
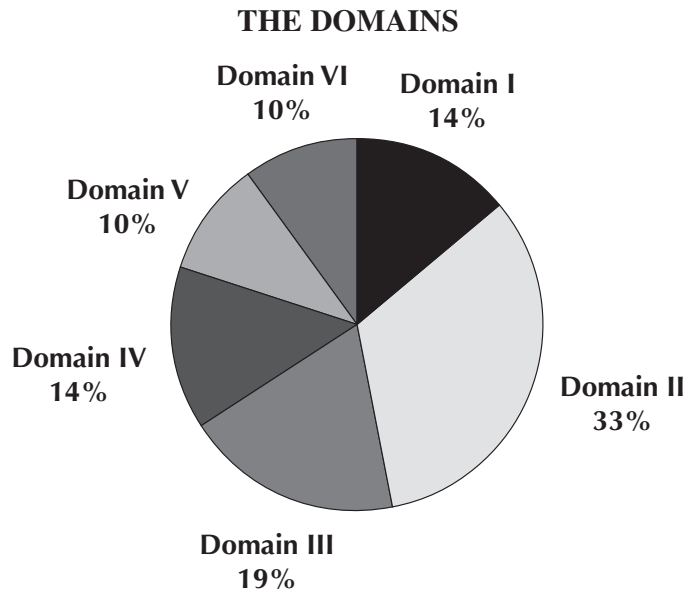
Domain III
19%

- **Domain I: Number Concepts**
  Standard Assessed: I

- **Domain II: Patterns and Algebra**
  Standard Assessed: II

- **Domain III: Geometry and Measurement**
  Standard Assessed: III

- **Domain IV: Probability and Statistics**
  Standard Assessed: IV

- **Domain V: Mathematical Processes and Perspectives**
  Standards Assessed: V and VI

- **Domain VI: Mathematical Learning, Instruction and Assessment**
  Standards Assessed: VII and VIII

TOTAL TEST BREAKDOWN

- Exam is offered as a paper-based or computer-administered test

- 90 Multiple-Choice Questions (80 Scored Questions*)

*Your final scaled score will be based only on scored questions.

THE STANDARDS

DOMAIN I — NUMBER CONCEPTS (approximately 14% of the test)

MATHEMATICS STANDARD I:

Number Concepts: The mathematics teacher understands and uses numbers, number systems and their structure, operations and algorithms, quantitative reasoning and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

DOMAIN II — PATTERNS AND ALGEBRA (approximately 33% of the test)

MATHEMATICS STANDARD II:

Patterns and Algebra: The mathematics teacher understands and uses patterns, relations, functions, algebraic reasoning, analysis and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

DOMAIN III — GEOMETRY AND MEASUREMENT (approximately 19% of the test)

MATHEMATICS STANDARD III:

Geometry and Measurement: The mathematics teacher understands and uses geometry, spatial reasoning, measurement concepts and principles and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

DOMAIN IV — PROBABILITY AND STATISTICS (approximately 14% of the test)

MATHEMATICS STANDARD IV:

Probability and Statistics: The mathematics teacher understands and uses probability and statistics, their applications and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

DOMAIN V — MATHEMATICAL PROCESSES AND PERSPECTIVES (approximately 10% of the test)

MATHEMATICS STANDARD V:

Mathematical Processes: The mathematics teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics and to communicate mathematically.

MATHEMATICS STANDARD VI:

Mathematical Perspectives: The mathematics teacher understands the historical development of mathematical ideas, the interrelationship between society and mathematics, the structure of mathematics and the evolving nature of mathematics and mathematical knowledge.

DOMAIN VI — MATHEMATICAL LEARNING, INSTRUCTION AND ASSESSMENT (approximately 10% of the test)

MATHEMATICS STANDARD VII:

Mathematical Learning and Instruction: The mathematics teacher understands how children learn and develop mathematical skills, procedures and concepts; knows typical errors students make; and uses this knowledge to plan, organize and implement instruction to meet curriculum goals and to teach all students to understand and use mathematics.

MATHEMATICS STANDARD VIII:

Mathematical Assessment: The mathematics teacher understands assessment, and uses a variety of formal and informal assessment techniques appropriate to the learner on an ongoing basis to monitor and guide instruction and to evaluate and report student progress.

COMPETENCIES

DOMAIN I — NUMBER CONCEPTS

COMPETENCY 001

THE TEACHER UNDERSTANDS THE REAL NUMBER SYSTEM AND ITS STRUCTURE, OPERATIONS, ALGORITHMS AND REPRESENTATIONS.

The beginning teacher:

A. Understands the concepts of place value, number base and decimal representations of real numbers.

B. Understands the algebraic structure and properties of the real number system and its subsets (e.g., real numbers as a field, integers as an additive group).

C. Describes and analyzes properties of subsets of the real numbers (e.g., closure, identities).

D. Selects and uses appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.

E. Uses a variety of models (e.g., geometric, symbolic) to represent operations, algorithms and real numbers.

F. Uses real numbers to model and solve a variety of problems.

G. Uses deductive reasoning to simplify and justify algebraic processes.

H. Demonstrates how some problems that have no solution in the integer or rational number systems have solutions in the real number system.

COMPETENCY 002

THE TEACHER UNDERSTANDS THE COMPLEX NUMBER SYSTEM AND ITS STRUCTURE, OPERATIONS, ALGORITHMS AND REPRESENTATIONS.

The beginning teacher:

A. Demonstrates how some problems that have no solution in the real number system have solutions in the complex number system.

B. Understands the properties of complex numbers (e.g., complex conjugate, magnitude/modulus, multiplicative inverse).

C. Understands the algebraic structure of the complex number system and its subsets (e.g., complex numbers as a field, complex addition as vector addition).

D. Selects and uses appropriate representations of complex numbers (e.g., vector, ordered pair, polar, exponential) for particular situations.

E. Describes complex number operations (e.g., addition, multiplication, roots) using symbolic and geometric representations.

COMPETENCY 003

THE TEACHER UNDERSTANDS NUMBER THEORY CONCEPTS AND PRINCIPLES AND USES NUMBERS TO MODEL AND SOLVE PROBLEMS IN A VARIETY OF SITUATIONS.

The beginning teacher:

A. Applies ideas from number theory (e.g., prime numbers and factorization, the Euclidean algorithm, divisibility, congruence classes, modular arithmetic, the fundamental theorem of arithmetic) to solve problems.

B. Applies number theory concepts and principles to justify and prove number relationships.

C. Compares and contrasts properties of vectors and matrices with properties of number systems (e.g., existence of inverses, non-commutative operations).

D. Uses properties of numbers (e.g., fractions, decimals, percents, ratios, proportions) to model and solve real-world problems.

E. Applies counting techniques such as permutations and combinations to quantify situations and solve problems.

F. Uses estimation techniques to solve problems and judges the reasonableness of solutions.

DOMAIN II — PATTERNS AND ALGEBRA

COMPETENCY 004

THE TEACHER USES PATTERNS TO MODEL AND SOLVE PROBLEMS AND FORMULATE CONJECTURES.

The beginning teacher:

A. Recognizes and extends patterns and relationships in data presented in tables, sequences or graphs.

B. Uses methods of recursion and iteration to model and solve problems.

C. Uses the principle of mathematical induction.

D. Analyzes the properties of sequences and series (e.g., Fibonacci, arithmetic, geometric) and uses them to solve problems involving finite and infinite processes.

E. Understands how sequences and series are applied to solve problems in the mathematics of finance (e.g., simple, compound and continuous interest rates; annuities).

COMPETENCY 005

THE TEACHER UNDERSTANDS ATTRIBUTES OF FUNCTIONS, RELATIONS AND THEIR GRAPHS.

The beginning teacher:

A. Understands when a relation is a function.

B. Identifies the mathematical domain and range of functions and relations and determines reasonable domains for given situations.

C. Understands that a function represents a dependence of one quantity on another and can be represented in a variety of ways (e.g., concrete models, tables, graphs, diagrams, verbal descriptions, symbols).

D. Identifies and analyzes even and odd functions, one-to-one functions, inverse functions and their graphs.

E. Applies basic transformations [e.g., $k f(x)$, $f(x) + k$, $f(x - k)$, $f(kx), |f(x)|$] to a parent function, f, and describes the effects on the graph of $y = f(x)$.

F. Performs operations (e.g., sum, difference, composition) on functions, finds inverse relations and describes results symbolically and graphically.

G. Uses graphs of functions to formulate conjectures of identities [e.g., $y = x^2 - 1$ and $y = (x - 1)(x + 1)$, $y = \log x^3$ and $y = 3 \log x$, $y = \sin\left(x + \dfrac{\pi}{2}\right)$ and $y = \cos x$].

COMPETENCY 006

THE TEACHER UNDERSTANDS LINEAR AND QUADRATIC FUNCTIONS, ANALYZES THEIR ALGEBRAIC AND GRAPHICAL PROPERTIES AND USES THEM TO MODEL AND SOLVE PROBLEMS.

The beginning teacher:

A. Understands the concept of slope as a rate of change and interprets the meaning of slope and intercept in a variety of situations.

B. Writes equations of lines given various characteristics (e.g., two points, a point and slope, slope and *y*-intercept).

C. Applies techniques of linear and matrix algebra to represent and solve problems involving linear systems.

D. Analyzes the zeros (real and complex) of quadratic functions.

E. Makes connections between the $y = ax^2 + bx + c$ and the $y = a(x - h)^2 + k$ representations of a quadratic function and its graph.

F. Solves problems involving quadratic functions using a variety of methods (e.g., factoring, completing the square, using the quadratic formula, using a graphing calculator).

G. Models and solves problems involving linear and quadratic equations and inequalities using a variety of methods, including technology.

COMPETENCY 007

THE TEACHER UNDERSTANDS POLYNOMIAL, RATIONAL, RADICAL, ABSOLUTE VALUE AND PIECEWISE FUNCTIONS, ANALYZES THEIR ALGEBRAIC AND GRAPHICAL PROPERTIES AND USES THEM TO MODEL AND SOLVE PROBLEMS.

The beginning teacher:

A. Recognizes and translates among various representations (e.g., written, tabular, graphical, algebraic) of polynomial, rational, radical, absolute value and piecewise functions.

B. Describes restrictions on the domains and ranges of polynomial, rational, radical, absolute value and piecewise functions.

C. Makes and uses connections among the significant points (e.g., zeros, local extrema, points where a function is not continuous or not differentiable) of a function, the graph of the function and the function's symbolic representation.

D. Analyzes functions in terms of vertical, horizontal and slant asymptotes.

E. Analyzes and applies the relationship between inverse variation and rational functions.

F.   Solves equations and inequalities involving polynomial, rational, radical, absolute value and piecewise functions using a variety of methods (e.g., tables, algebraic methods, graphs, use of a graphing calculator) and evaluates the reasonableness of solutions.

G.   Models situations using polynomial, rational, radical, absolute value and piecewise functions and solves problems using a variety of methods, including technology.

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COMPETENCY 008

THE TEACHER UNDERSTANDS EXPONENTIAL AND LOGARITHMIC FUNCTIONS, ANALYZES THEIR ALGEBRAIC AND GRAPHICAL PROPERTIES AND USES THEM TO MODEL AND SOLVE PROBLEMS.

The beginning teacher:

A.   Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of exponential and logarithmic functions.

B.   Recognizes and uses connections among significant characteristics (e.g., intercepts, asymptotes) of a function involving exponential or logarithmic expressions, the graph of the function and the function's symbolic representation.

C.   Understands the relationship between exponential and logarithmic functions and uses the laws and properties of exponents and logarithms to simplify expressions and solve problems.

D.   Uses a variety of representations and techniques (e.g., numerical methods, tables, graphs, analytic techniques, graphing calculators) to solve equations, inequalities and systems involving exponential and logarithmic functions.

E.   Models and solves problems involving exponential growth and decay.

F.   Uses logarithmic scales (e.g., Richter, decibel) to describe phenomena and solve problems.

G.   Uses exponential and logarithmic functions to model and solve problems involving the mathematics of finance (e.g., compound interest).

H.   Uses the exponential function to model situations and solve problems in which the rate of change of a quantity is proportional to the current amount of the quantity [i.e., $f'(x) = k f(x)$].

COMPETENCY 009

THE TEACHER UNDERSTANDS TRIGONOMETRIC AND CIRCULAR FUNCTIONS, ANALYZES THEIR ALGEBRAIC AND GRAPHICAL PROPERTIES AND USES THEM TO MODEL AND SOLVE PROBLEMS.

The beginning teacher:

A. Analyzes the relationships among the unit circle in the coordinate plane, circular functions and the trigonometric functions.

B. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of trigonometric functions and their inverses.

C. Recognizes and uses connections among significant properties (e.g., zeros, axes of symmetry, local extrema) and characteristics (e.g., amplitude, frequency, phase shift) of a trigonometric function, the graph of the function and the function's symbolic representation.

D. Understands the relationships between trigonometric functions and their inverses and uses these relationships to solve problems.

E. Uses trigonometric identities to simplify expressions and solve equations.

F. Models and solves a variety of problems (e.g., analyzing periodic phenomena) using trigonometric functions.

G. Uses graphing calculators to analyze and solve problems involving trigonometric functions.

COMPETENCY 010

THE TEACHER UNDERSTANDS AND SOLVES PROBLEMS USING DIFFERENTIAL AND INTEGRAL CALCULUS.

The beginning teacher:

A. Understands the concept of limit and the relationship between limits and continuity.

B. Relates the concept of average rate of change to the slope of the secant line and relates the concept of instantaneous rate of change to the slope of the tangent line.

C. Uses the first and second derivatives to analyze the graph of a function (e.g., local extrema, concavity, points of inflection).

D. Understands and applies the fundamental theorem of calculus and the relationship between differentiation and integration.

E. Models and solves a variety of problems (e.g., velocity, acceleration, optimization, related rates, work, center of mass) using differential and integral calculus.

F. Analyzes how technology can be used to solve problems and illustrate concepts involving differential and integral calculus.

DOMAIN III — GEOMETRY AND MEASUREMENT

COMPETENCY 011

THE TEACHER UNDERSTANDS MEASUREMENT AS A PROCESS.

The beginning teacher:

A. Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.

B. Applies formulas for perimeter, area, surface area and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.

C. Recognizes the effects on length, area or volume when the linear dimensions of plane figures or solids are changed.

D. Applies the Pythagorean theorem, proportional reasoning and right triangle trigonometry to solve measurement problems.

E. Relates the concept of area under a curve to the limit of a Riemann sum.

F. Uses integral calculus to compute various measurements associated with curves and regions (e.g., area, arc length) in the plane, and measurements associated with curves, surfaces and regions in three-space.

COMPETENCY 012

THE TEACHER UNDERSTANDS GEOMETRIES, IN PARTICULAR EUCLIDEAN GEOMETRY, AS AXIOMATIC SYSTEMS.

The beginning teacher:

A. Understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).

B. Uses properties of points, lines, planes, angles, lengths and distances to solve problems.

C. Applies the properties of parallel and perpendicular lines to solve problems.

D. Uses properties of congruence and similarity to explore geometric relationships, justify conjectures and prove theorems.

E. Describes and justifies geometric constructions made using compass and straightedge, reflection devices and other appropriate technologies.

F. Demonstrates an understanding of the use of appropriate software to explore attributes of geometric figures and to make and evaluate conjectures about geometric relationships.

G. Compares and contrasts the axioms of Euclidean geometry with those of non-Euclidean geometry (i.e., hyperbolic and elliptic geometry).

COMPETENCY 013

THE TEACHER UNDERSTANDS THE RESULTS, USES AND APPLICATIONS OF EUCLIDEAN GEOMETRY.

The beginning teacher:

A. Analyzes the properties of polygons and their components.

B. Analyzes the properties of circles and the lines that intersect them.

C. Uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).

D. Computes the perimeter, area and volume of figures and shapes created by subdividing and combining other figures and shapes (e.g., arc length, area of sectors).

E. Analyzes cross-sections and nets of three-dimensional shapes.

F. Uses top, front, side and corner views of three-dimensional shapes to create complete representations and solve problems.

G. Applies properties of two- and three-dimensional shapes to solve problems across the curriculum and in everyday life.

COMPETENCY 014

THE TEACHER UNDERSTANDS COORDINATE, TRANSFORMATIONAL AND VECTOR GEOMETRY AND THEIR CONNECTIONS.

The beginning teacher:

A. Identifies transformations (i.e., reflections, translations, glide-reflections, rotations, dilations) and explores their properties.

B. Uses the properties of transformations and their compositions to solve problems.

C. Uses transformations to explore and describe reflectional, rotational and translational symmetry.

D. Applies transformations in the coordinate plane.

E. Applies concepts and properties of slope, midpoint, parallelism, perpendicularity and distance to explore properties of geometric figures and solve problems in the coordinate plane.

F. Uses coordinate geometry to derive and explore the equations, properties and applications of conic sections (i.e., lines, circles, hyperbolas, ellipses, parabolas).

G. Relates geometry and algebra by representing transformations as matrices and uses this relationship to solve problems.

H. Explores the relationship between geometric and algebraic representations of vectors and uses this relationship to solve problems.

DOMAIN IV — PROBABILITY AND STATISTICS

COMPETENCY 015

THE TEACHER UNDERSTANDS HOW TO USE APPROPRIATE GRAPHICAL AND NUMERICAL TECHNIQUES TO EXPLORE DATA, CHARACTERIZE PATTERNS AND DESCRIBE DEPARTURES FROM PATTERNS.

The beginning teacher:

A. Selects and uses an appropriate measurement scale (i.e., nominal, ordinal, interval, ratio) to answer research questions and analyze data.

B. Organizes, displays and interprets data in a variety of formats (e.g., tables, frequency distributions, scatter plots, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).

C. Applies concepts of center, spread, shape and skewness to describe a data distribution.

D. Understands measures of central tendency (i.e., mean, median, mode) and dispersion (i.e., range, interquartile range, variance, standard deviation).

E. Applies linear transformations (i.e., translating, stretching, shrinking) to convert data and describes the effect of linear transformations on measures of central tendency and dispersion.

F. Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers and measures of central tendency and dispersion.

G. Supports arguments, makes predictions and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.

COMPETENCY 016

THE TEACHER UNDERSTANDS CONCEPTS AND APPLICATIONS OF PROBABILITY.

The beginning teacher:

A. Understands how to explore concepts of probability through sampling, experiments and simulations and generates and uses probability models to represent situations.

B. Uses the concepts and principles of probability to describe the outcomes of simple and compound events.

C. Determines probabilities by constructing sample spaces to model situations.

D. Solves a variety of probability problems using combinations and permutations.

E. Solves a variety of probability problems using ratios of areas of geometric regions.

F. Calculates probabilities using the axioms of probability and related theorems and concepts such as the addition rule, multiplication rule, conditional probability and independence.

G. Understands expected value, variance and standard deviation of probability distributions (e.g., binomial, geometric, uniform, normal).

H. Applies concepts and properties of discrete and continuous random variables to model and solve a variety of problems involving probability and probability distributions (e.g., binomial, geometric, uniform, normal).

COMPETENCY 017

THE TEACHER UNDERSTANDS THE RELATIONSHIPS AMONG PROBABILITY THEORY, SAMPLING AND STATISTICAL INFERENCE AND HOW STATISTICAL INFERENCE IS USED IN MAKING AND EVALUATING PREDICTIONS.

The beginning teacher:

A. Applies knowledge of designing, conducting, analyzing and interpreting statistical experiments to investigate real-world problems.

B. Analyzes and interprets statistical information (e.g., the results of polls and surveys) and recognizes misleading as well as valid uses of statistics.

C. Understands random samples and sample statistics (e.g., the relationship between sample size and confidence intervals, biased or unbiased estimators).

D. Makes inferences about a population using binomial, normal and geometric distributions.

E. Describes and analyzes bivariate data using various techniques (e.g., scatterplots, regression lines, outliers, residual analysis, correlation coefficients).

F. Understands how to transform nonlinear data into linear form in order to apply linear regression techniques to develop exponential, logarithmic and power regression models.

G. Uses the law of large numbers and the central limit theorem in the process of statistical inference.

H. Estimates parameters (e.g., population mean and variance) using point estimators (e.g., sample mean and variance).

I. Understands principles of hypotheses testing.

DOMAIN V — MATHEMATICAL PROCESSES AND PERSPECTIVES

COMPETENCY 018

THE TEACHER UNDERSTANDS MATHEMATICAL REASONING AND PROBLEM SOLVING.

The beginning teacher:

A. Understands the nature of proof, including indirect proof, in mathematics.

B. Applies correct mathematical reasoning to derive valid conclusions from a set of premises.

C. Uses inductive reasoning to make conjectures and uses deductive methods to evaluate the validity of conjectures.

D. Uses formal and informal reasoning to justify mathematical ideas.

E. Understands the problem-solving process (i.e., recognizing that a mathematical problem can be solved in a variety of ways, selecting an appropriate strategy, evaluating the reasonableness of a solution).

F. Evaluates how well a mathematical model represents a real-world situation.

COMPETENCY 019

THE TEACHER UNDERSTANDS MATHEMATICAL CONNECTIONS BOTH WITHIN AND OUTSIDE OF MATHEMATICS AND HOW TO COMMUNICATE MATHEMATICAL IDEAS AND CONCEPTS.

The beginning teacher:

A. Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of a circle as a quadratic function of the radius, probability as the ratio of two areas, area of a plane region as a definite integral).

B. Understands how mathematics is used to model and solve problems in other disciplines (e.g., art, music, science, social science, business).

C. Translates mathematical ideas between verbal and symbolic forms.

D. Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, concrete).

E. Understands the use of visual media, such as graphs, tables, diagrams and animations, to communicate mathematical information.

F. Uses appropriate mathematical terminology to express mathematical ideas.

DOMAIN VI — MATHEMATICAL LEARNING, INSTRUCTION AND ASSESSMENT

COMPETENCY 020

THE TEACHER UNDERSTANDS HOW CHILDREN LEARN MATHEMATICS AND PLANS, ORGANIZES AND IMPLEMENTS INSTRUCTION USING KNOWLEDGE OF STUDENTS, SUBJECT MATTER AND STATEWIDE CURRICULUM (TEXAS ESSENTIAL KNOWLEDGE AND SKILLS [TEKS]).

The beginning teacher:

A. Applies research-based theories of learning mathematics to plan appropriate instructional activities for all students.

B. Understands how students differ in their approaches to learning mathematics.

C. Uses students' prior mathematical knowledge to build conceptual links to new knowledge and plans instruction that builds on students' strengths and addresses students' needs.

D. Understands how learning may be enhanced through the use of manipulatives, technology and other tools (e.g., stop watches, scales, rulers).

E. Understands how to provide instruction along a continuum from concrete to abstract.

F. Understands a variety of instructional strategies and tasks that promote students' abilities to do the mathematics described in the TEKS.

G. Understands how to create a learning environment that provides all students, including English-language learners, with opportunities to develop and improve mathematical skills and procedures.

H. Understands a variety of questioning strategies to encourage mathematical discourse and to help students analyze and evaluate their mathematical thinking.

I. Understands how to relate mathematics to students' lives and to a variety of careers and professions.

COMPETENCY 021

THE TEACHER UNDERSTANDS ASSESSMENT AND USES A VARIETY OF FORMAL AND INFORMAL ASSESSMENT TECHNIQUES TO MONITOR AND GUIDE MATHEMATICS INSTRUCTION AND TO EVALUATE STUDENT PROGRESS.

The beginning teacher:

A. Understands the purpose, characteristics and uses of various assessments in mathematics, including formative and summative assessments.

B. Understands how to select and develop assessments that are consistent with what is taught and how it is taught.

C. Understands how to develop a variety of assessments and scoring procedures consisting of worthwhile tasks that assess mathematical understanding, common misconceptions and error patterns.

D. Understands the relationship between assessment and instruction and knows how to evaluate assessment results to design, monitor and modify instruction to improve mathematical learning for all students, including English-language learners.

Chapter 4

Succeeding on Multiple-Choice Questions

★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★

APPROACHES TO ANSWERING MULTIPLE-CHOICE QUESTIONS

The purpose of this section is to describe multiple-choice question formats that you will see on the Mathematics 8–12 test and to suggest possible ways to approach thinking about and answering the multiple-choice questions. However, these approaches are not intended to replace familiar test-taking strategies with which you are already comfortable and that work for you.

The Mathematics 8–12 test is designed to include a total of 90 multiple-choice questions, out of which 80 are scored. Your final scaled score will be based only on scored questions. The questions that are not scored are being pilot tested in order to collect information about how these questions will perform under actual testing conditions. These questions are not identified on the test.

All multiple-choice questions on this test are designed to assess your knowledge of the content described in the test framework. In most cases, you are expected to demonstrate more than just your ability to recall factual information. You may be asked to solve a multi-step problem; analyze and interpret mathematical information in a variety of formats; determine a mathematical function that models a given situation; or supply information needed to prove a mathematical statement.

When you are ready to respond to a multiple-choice question, you must choose one of four answer options labeled A, B, C and D. Leave no questions unanswered. Nothing is subtracted from your score if you answer a question incorrectly. Questions for which you mark no answer or more than one answer are not counted in scoring. Your score will be determined by the number of questions for which you select the best answer.

Calculators. Some test questions for Mathematics 8–12 are designed to be solved with a graphing calculator. If you want to use a calculator, you must bring your own calculator to the test administration. Only the brands and models listed in the current TExES *Registration Bulletin* may be used. All calculators on the approved list are graphing calculators. Graphing calculators perform all the operations of typical scientific calculators. Test administration staff will clear the memory of your calculator both before and after the test. Sharing of calculators will not be permitted. The approved calculator brands and models are subject to change. If there is a change, examinees will be notified.

Definitions and Formulas. A set of definitions and formulas will be provided. A copy of those definitions and formulas is also provided in Chapter 5 of this preparation manual.

QUESTION FORMATS

You may see the following types of multiple-choice questions on the test.

— Single Questions
— Questions with Stimulus Material
— Clustered Questions

On the following pages, you will find descriptions of these commonly used question formats, along with suggested approaches for responding to each type of question. In the actual testing situation, if you are taking the paper-based version of the test, you may mark the test questions and/or write in the margins of your test booklet. **Your final response must be indicated on the answer sheet provided.** If you are taking the test via computer, you may write on the scratch paper provided at the testing center. **Your final response must be selected on the computer.**

SINGLE QUESTIONS

In the single-question format, a problem is presented as a direct question or an incomplete statement, and four answer options appear below the question. The following question is an example of this type. It tests knowledge of Mathematics 8–12 Competency 010: *The teacher understands and solves problems using differential and integral calculus.*

EXAMPLE

Use the diagram below to answer the question that follows.
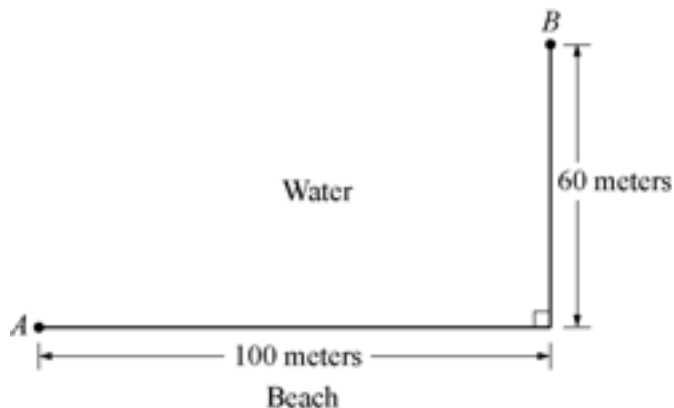
A lifeguard sitting on a beach at point *A* sees a swimmer in distress at point *B*. The lifeguard can run at a rate of 3 meters per second and can swim at a rate of 1.5 meters per second. To minimize the amount of time it takes to reach the swimmer, how far along the beach should the lifeguard run before entering the water?

    A.   40 meters

    B.   65 meters

    C.   73 meters

    D. 100 meters

SUGGESTED APPROACH

Read the question carefully and critically. Think about what it is asking and the situation it is describing. Eliminate any obviously wrong answers, select the correct answer choice and mark your answer.

In analyzing this problem, redrawing the diagram to highlight the important information may be helpful.

4
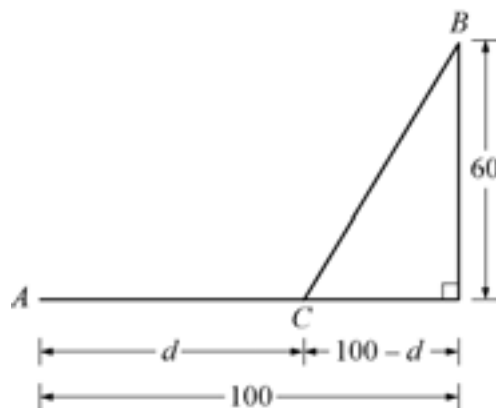
Let d represent the distance in meters that the lifeguard runs along the beach. Then by an application of the Pythagorean theorem, the distance traveled in water is represented by $\sqrt{60^2 + (100 - d)^2}$.

Since *distance = rate × time* and the lifeguard can run at 3 meters per second and swim at 1.5 meters per second, the time it takes the lifeguard to run along the beach, t_b, can be represented by $\frac{d}{3}$, and

the time it takes the lifeguard to swim in the water, t_w, can be represented by $\frac{\sqrt{60^2 + (100 - d)^2}}{1.5}$. Thus the total time, t, it takes the lifeguard to travel to the swimmer can be represented by $t_b + t_w$.

To solve the problem, we need to find the value of d that minimizes the function

$t = t_b + t_w = \frac{d}{3} + \frac{\sqrt{60^2 + (100 - d)^2}}{1.5}$. This can be done using either differential calculus or a graphing

approach. We will use a graphing approach. A graphing calculator can be used to produce a graph similar to the one on the following page.
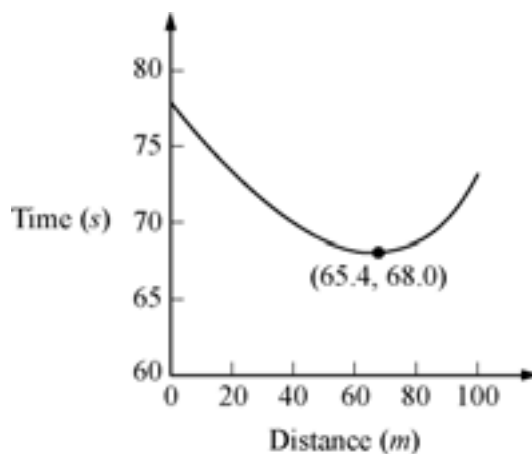
Using the capabilities of the calculator, you see that the minimum value of the function t occurs when d is approximately 65 meters, or option B.

Option A results from dividing 60 by 1.5, which is the time required to swim 60 meters. Option C results from misusing parentheses when entering the equation for t into the graphing utility; i.e., entering $\sqrt{\dfrac{60^2 + (100 - d)^2}{1.5}}$ instead of $\dfrac{\sqrt{60^2 + (100 - d)^2}}{1.5}$. Option D results from minimizing the

function $t_w = \dfrac{\sqrt{60^2 + (100 - d)^2}}{1.5}$ instead of the expression for t, the total time required to reach the

swimmer.

QUESTIONS WITH STIMULUS MATERIAL

Some questions on this test are preceded by stimulus material that relates to the question. Some types of stimulus material included on the test are graphs of one or more mathematical functions, geometric diagrams, charts, data tables, equations and descriptions of classroom situations. In such cases, you will generally be given information followed by an event to analyze, a problem to solve or a decision to make.

You can use several different approaches to respond to these types of questions. Some commonly used strategies are listed below.

Strategy 1 Skim the stimulus material to understand its purpose, its arrangement and/or its content. Then read the question and refer again to the stimulus material to obtain the specific information you need to answer the question.

Strategy 2 Read the question *before* considering the stimulus material. The theory behind this strategy is that the content of the question will help you identify the purpose of the stimulus material and locate the information you need to answer the question.

Strategy 3 Use a combination of both strategies; apply the "read the stimulus first" strategy with shorter, more familiar stimuli and the "read the question first" strategy with longer, more complex or less familiar stimuli. You can experiment with the sample questions in this manual and then use the strategy with which you are most comfortable when you take the actual test.

Whether you read the stimulus before or after you read the question, you should read it carefully and critically. If you are taking a paper-based test, you may want to underline its important points to help you answer the question.

As you consider questions set in educational contexts, try to enter into the identified teacher's frame of mind and use that teacher's point of view to answer the questions that accompany the stimulus. Be sure to consider the questions in terms of only the information provided in the stimulus — not in terms of your own class experiences or individual students you may have known.

EXAMPLE

First read the stimulus (a learning expectation from the statewide mathematics curriculum).

Use the student expectation below from the Texas Essential Knowledge and Skills (TEKS) to answer the two questions that follow.

The student uses characteristics of the quadratic parent function to sketch the related graphs and makes connections between the $y = ax^2 + bx + c$ and the $y = a(x - h)^2 + k$ symbolic representations of quadratic functions.

Now you are prepared to address the first of the two questions associated with this stimulus. The first question measures Mathematics 8–12, Competency 020: *The teacher understands how children learn mathematics and plans, organizes and implements instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).*

1. Which of the following algebraic techniques will students need to know to symbolically convert a quadratic function of the form $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$?

   A. Solving systems of equations

   B. Completing the square

   C. Solving quadratic equations

   D. Simplifying polynomial expressions

SUGGESTED APPROACH

You are asked to identify the algebraic technique that students should use to convert the expression $y = ax^2 + bx + c$ into the expression $y = a(x - h)^2 + k$. The following steps show how this conversion can be achieved.

First rewrite the expression $y = ax^2 + bx + c$ as $y = a\left(x^2 + \dfrac{b}{a}x\right) + c$ by factoring a from the quantity $ax^2 + bx$. Next, take one-half the coefficient of the linear term, square it, and add this quantity inside the parentheses while adding the product of the quantity's additive inverse and a outside of the parentheses. Note that this is equivalent to adding $\dfrac{b^2}{4a}$ and $-\dfrac{b^2}{4a}$ to the same side of the equation as follows:

$$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a}$$

Notice that the quantity inside the parentheses is a perfect square and can be factored.

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

This expression is equivalent to $y = a(x - h)^2 + k$, with $h = -\dfrac{b}{2a}$ and $k = \dfrac{4ac - b^2}{4a}$, which are the x- and y-coordinates of the vertex of the graph of $y = ax^2 + bx + c$. This algebraic method of converting the first expression into the second is known as completing the square. Therefore, **option B is correct**.

Option A, solving systems of equations, is not helpful in this situation because the student is being asked to rewrite an equation, not solve it. Option C is incorrect because the student is being asked to rewrite a quadratic equation, not solve it. Finally, although one can simplify the expression $y = a(x - h)^2 + k$ and compare it to $y = ax^2 + bx + c$, this approach is ineffective when applied in the opposite direction, which makes option D incorrect.

Now you are ready to answer the next question. The second question measures Competency 021: *The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.*

2. Which of the following exercises best assesses student understanding of the expectation from the statewide curriculum (TEKS).

A. Use a graphing calculator to graph the function $y = x^2 - 4x + 3$, and use the graph to find the zeros of the function.

B. Write a real-world word problem that is modeled by the function $y = x^2 - 4x + 3$, and relate the zeros of the function to the graph of $y = x^2 - 4x + 3$.

C. Describe how the graph of $y = (x - 3)(x - 1)$ is related to the graph of $y = x^2 - 4x + 3$.

D. Describe how the graph of $y = x^2$ is related to the graph of $y = x^2 - 4x + 3$.

SUGGESTED APPROACH

You are asked to select an activity that would best assess student understanding of converting a function of the form $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$ and then analyzing the graph of this function in relation to the quadratic parent function $y = x^2$. Carefully read each of the responses to determine how well they assess student understanding of this topic.

Option A asks the student to enter a quadratic function into a graphing calculator and then use the capabilities of the graphing calculator to estimate the zeros of the function. This is a method of using technology to solve a quadratic equation, and hence is incorrect.

Option B asks the student to create a problem that can be modeled by a specific quadratic equation and to relate the graph of the equation to the problem. This assessment would be useful for evaluating student understanding of applications of quadratic functions, but not for assessing understanding of the two different symbolic representations of the quadratic function. Option B is therefore incorrect.

Option C assesses understanding of the fact that a factored quadratic function has the same graph as the expanded, or unfactored, quadratic function. Option C would not assess the given learning expectation and is therefore incorrect.

Option D assesses student understanding of how the graph of $y = x^2$ is related to that of a more complicated quadratic function involving a linear term and a constant term. Expressing the function $y = x^2 - 4x + 3$ in the form $y = (x - 2)^2 - 1$ allows a student to determine by inspection that the vertex is at (2, –1). This implies that the graph of $y = x^2 - 4x + 3$ can be obtained by translating the graph of $y = x^2$ two units in the positive *x*-direction and one unit in the negative *y*-direction.

This analysis of the four choices should lead you to select **option D as the best response**.

CLUSTERED QUESTIONS

You may have one or more questions related to a single stimulus. When you have two or more questions related to a single stimulus, the group of questions is called a cluster.

Chapter 5

Multiple-Choice Practice Questions

★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★
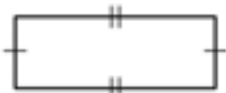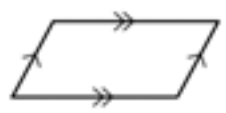
SAMPLE MULTIPLE-CHOICE QUESTIONS

This section presents some sample test questions for you to review as part of your preparation for the test. To demonstrate how each competency may be assessed, each sample question is accompanied by the competency that it measures. While studying, you may wish to read the competency before and after you consider each sample question. Please note that the competency statements will not appear on the actual test.

An answer key follows the sample questions. The answer key lists the question number and correct answer for each sample test question. Please note that the answer key also lists the competency assessed by each question and that the sample questions are not necessarily presented in competency order.

The sample questions are included to illustrate the formats and types of questions you will see on the test; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual test.

5

Definitions and Formulas for Mathematics 8-12

CALCULUS

First Derivative: $f'(x) = \dfrac{dy}{dx}$

Second Derivative: $f''(x) = \dfrac{d^2y}{dx^2}$

PROBABILITY

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$

ALGEBRA

i $\qquad\qquad$ $i^2 = -1$

A^{-1} $\qquad\qquad$ inverse of matrix A

$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

Compound interest,
where A is the final value
P is the principal
r is the interest rate
t is the term
n is the number of
divisions within
the term

$[x] = n$ $\qquad$ Greatest integer function,
where n is the integer such
that $n \leq x < n + 1$

GEOMETRY
Congruent Angles

Congruent Sides

Parallel Sides

Circumference of a Circle
$$C = 2\pi r$$

VOLUME

Cylinder: (area of base) × height

Cone: $\dfrac{1}{3}$ (area of base) × height

Sphere: $\dfrac{4}{3}\pi r^3$

Prism: (area of base) × height

AREA

Triangle: $\dfrac{1}{2}$ (base × height)

Rhombus: $\dfrac{1}{2}$ (diagonal$_1$ × diagonal$_2$)

Trapezoid: $\dfrac{1}{2}$ height (base$_1$ + base$_2$)

Sphere: $4\pi r^2$

Circle: πr^2

Lateral surface area of cylinder: $2\pi rh$

TRIGONOMETRY

Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Law of Cosines:
$c^2 = a^2 + b^2 - 2ab \cos C$
$b^2 = a^2 + c^2 - 2ac \cos B$
$a^2 = b^2 + c^2 - 2bc \cos A$
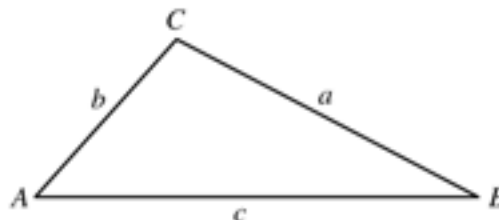
END OF DEFINITIONS AND FORMULAS

5

COMPETENCY 001

1. **Use the addition problem below to answer the question that follows.**

$$
\begin{array}{r}
12 \\
29 \\
88 \\
+\ 11 \\
\hline
\end{array}
$$

When given the addition problem above, a student quickly said "140." When asked how she solved the problem, the student replied, "I added 88 and 12 to get 100, and 29 and 11 to get 40. Then I added these two numbers together." Which of the following two properties of addition did the student use in solving this problem?

A. Associative and commutative

B. Associative and additive identity

C. Commutative and additive identity

D. Distributive and additive inverse

COMPETENCY 001

2. A student starts with a positive number, x, and takes the positive square root four times in succession. The final result is equivalent to which of the following?

A. $x^{0.25}$

B. $\left(\sqrt{x}\right)^{1/3}$

C. $\left(\sqrt{x}\right)^{1/4}$

D. $x^{1/16}$

COMPETENCY 002

3.   **Use the figure below to answer the question that follows.**
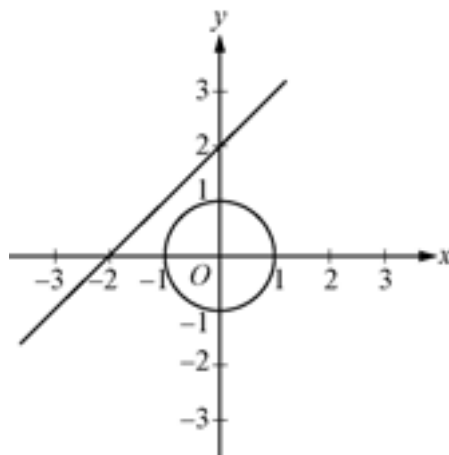
The figure above shows a line and a circle in the *xy*-plane. Which of the following is a possible value of *x* in the set of complex numbers such that the pair (x, y), where *y* is also a complex number, is a solution to the system of equations whose graphs are shown above?

A. $-2 + \sqrt{2}\, i$

B. $-1 + \dfrac{1}{2} i$

C. $-1 + \dfrac{\sqrt{2}}{2} i$

D. $-1 + 2\sqrt{2}\, i$

COMPETENCY 002

4.   Two complex numbers, z_1 and z_2, are expressed in polar form as follows:

$$z_1 = \left(\sqrt{2}, 10°\right) \text{ and } z_2 = \left(2\sqrt{2}, 15°\right)$$

Which of the following is the product of z_1 and z_2 expressed in polar form?

A. $\left(3\sqrt{2}, 25°\right)$

B. $(4, 25°)$

C. $\left(3\sqrt{2}, 150°\right)$

D. $(4, 150°)$

COMPETENCY 003

5.   **Use the theorem below to answer the question that follows.**

> If the sum of the digits of a positive integer is divisible
> by 3, then the integer itself is divisible by 3.

Which of the following ways of expressing a positive 3-digit integer, n, best illustrates why the theorem stated above is true?

A. $n = a\left(10^2\right) + b(10) + c$

B. $n = a(100) + b(10) + c$

C. $n = a(99 + 1) + b(9 + 1) + c$

D. $n = a(300 - 200) + b(30 - 20) + c(9 - 8)$

COMPETENCY 003

6.   Which of the following statements about real numbers and $n \times n$ real matrices is true?

A. The product of two real numbers is a real number, but the product of two $n \times n$ real matrices might not be an $n \times n$ real matrix.

B. All real numbers have an additive inverse, but some $n \times n$ real matrices do not.

C. Multiplication of real numbers is commutative, but multiplication of $n \times n$ real matrices is not.

D. The set of real numbers form a group under addition, but the set of $n \times n$ real matrices do not.

COMPETENCY 004

7.   **Use the diagram below to answer the question that follows.**

1   2   3   4
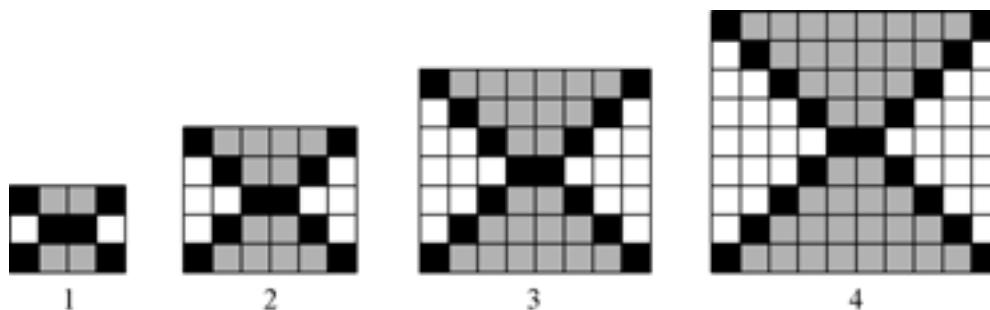
The first four elements of a pattern are shown in the diagram above. If the pattern continues, how many black boxes will there be in the twelfth element of the pattern?

A. 46

B. 48

C. 50

D. 52

COMPETENCY 004

8.   **Use the diagram below to answer the question that follows.**

In the sequence of rectangles shown above, the first (left-most) rectangle is a square of area 1. Each successive rectangle has the same base but has two-thirds the height of the previous rectangle. If this pattern is continued indefinitely, which of the following series represents the sum of the areas of all the rectangles?

A. $1 + \dfrac{1}{3} + \dfrac{1}{6} + \dfrac{1}{12} + \dfrac{1}{24} + \cdots$

B. $1 + \dfrac{2}{3} + \dfrac{4}{6} + \dfrac{6}{9} + \dfrac{8}{12} + \cdots$

C. $1 + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \dfrac{1}{81} + \cdots$

D. $1 + \dfrac{2}{3} + \dfrac{4}{9} + \dfrac{8}{27} + \dfrac{16}{81} + \cdots$

COMPETENCY 005

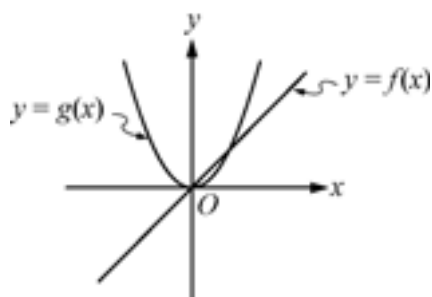9. **Use the diagram below to answer the question that follows.**
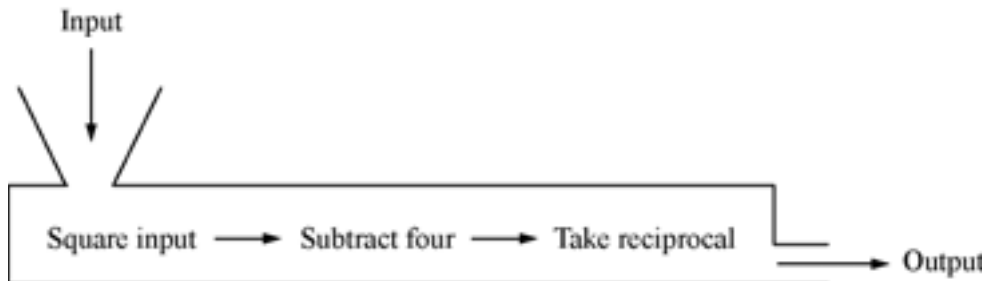
The diagram above represents a function. Which of the following could be the domain of the function?

A. All real numbers

B. All real numbers greater than –4

C. All real numbers except 2 and –2
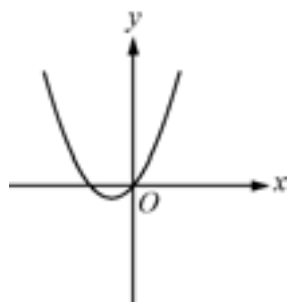
D. All real numbers except 4 and –4

COMPETENCY 005

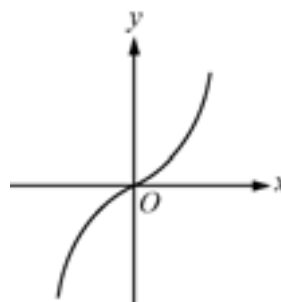10. **Use the graphs of the functions below to answer the question that follows.**

Given the graphs of the linear and quadratic functions above, which of the following graphs represents the sum of the two functions?
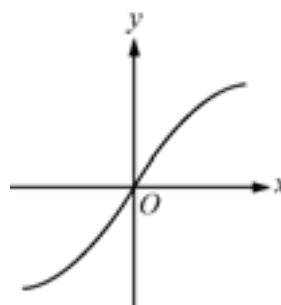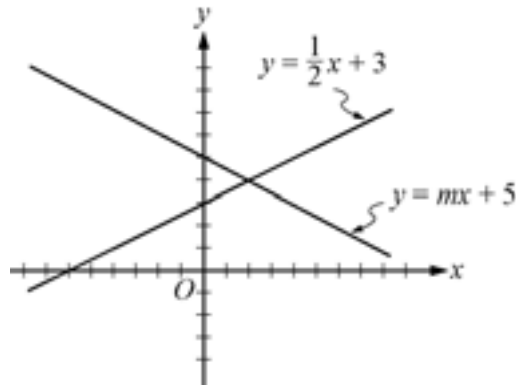
5

A.

C.

B.
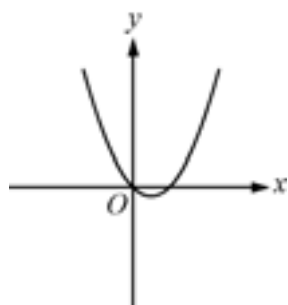
D.

COMPETENCY 006

11. **Use the figure below to answer the question that follows.**

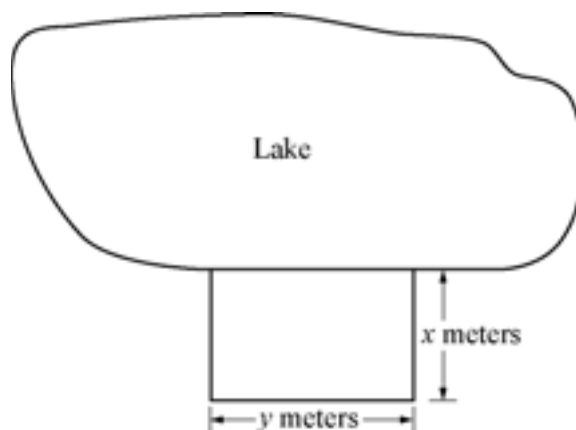The lines above represent a system of linear equations. For what values of m will the solution to the system be in the first quadrant?

A. $m < \dfrac{1}{2}$

B. $m < 5$

C. $m > \dfrac{1}{2}$

D. $m > 5$

COMPETENCY 006

12. **Use the figure below to answer the question that follows.**

A person is building a fence along three sides of a rectangular plot and using the straight part of a lake shoreline as the fourth side of the plot, as shown above. No fencing is required for the fourth side. The person has 110 meters of fencing, and the area of the plot is 1200 square meters. Which of the following quadratic equations could be solved to find the smaller dimension, x, of the plot?

A. $x^2 + 110x + 1200 = 0$

B. $x^2 - 55x + 600 = 0$

C. $x^2 - 55x + 1200 = 0$

D. $x^2 - 110x + 600 = 0$

COMPETENCY 007

13. An engineer's solution to a problem requires a nonzero polynomial function, f, of lowest degree that passes through the points $(-4, 0)$, $(-2, 0)$, $(1, 0)$, and $(3, 0)$. Which of the following functions satisfies the engineer's requirement?

A. $f(x) = (x - 4)(x - 2)(x + 1)(x + 3)$

B. $f(x) = (x + 4)(x + 2) + (x - 1)(x - 3)$

C. $f(x) = (x - 4)(x - 2) + (x + 1)(x + 3)$

D. $f(x) = (x + 4)(x + 2)(x - 1)(x - 3)$

COMPETENCY 007

14. The function f defined by $f(x) = \dfrac{8x^2 - x + 4}{x^2 - 6x + 5}$ has how many asymptotes?

   A. None

   B. One

   C. Two

   D. Three

COMPETENCY 008

15. The active ingredient in a chemical solution used to control insects decays exponentially with time. The active ingredient decays by 50 percent 12 hours after application. What percent of the active ingredient decays 36 hours after application?

   A. 65.0%

   B. 74.0%

   C. 83.0%

   D. 87.5%

COMPETENCY 008

16. The population of a mold, $P(t)$, is modeled by an exponential function of the form $P(t) = Ae^{bt}$. The table below gives the population at two different times.
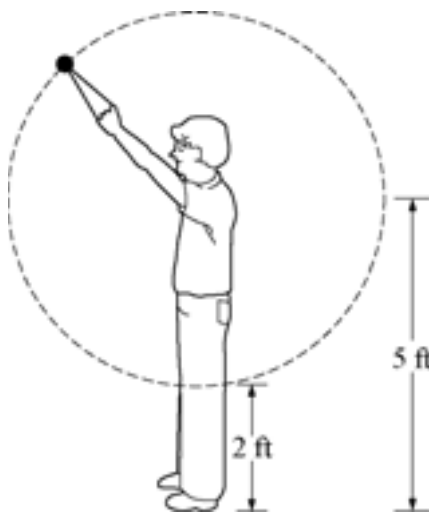
Time (hours)	Population Size
0	150
10	450

   What is the value of b ?

   A. $\ln 3$

   B. $\dfrac{\ln 3}{10}$

   C. $\ln \dfrac{1}{3}$

   D. $\ln 10 - \ln 3$

COMPETENCY 009

17. **Use the diagram below to answer the question that follows.**

A person swings a small ball attached to a string around in a circle of radius 3 feet at a constant angular speed of 300° per second. If the closest distance the ball comes to the ground is 2 feet, which of the following functions best models the height h, in feet, of the ball aboveground as a function of time t, in seconds?

A. $h = 3 \sin 300t + 2$

B. $h = 6 \sin 300t + 5$

C. $h = 3 \sin 300t + 5$

D. $h = 6 \sin 300t + 2$

COMPETENCY 009

18. The mean air temperature for a city is modeled by the function

$$T(d) = 45 \sin\left(\frac{2\pi}{365}(d - 108)\right) + 30$$

where T is the temperature, in degrees Fahrenheit (°F), and d is the time, in days, with $d = 1$ representing January 1. For which values of d is the mean air temperature in the city at least 60°F ?

A. 132–266

B. 138–260

C. 144–254

D. 150–248

COMPETENCY 010

19. **Use the figure below to answer the question that follows.**
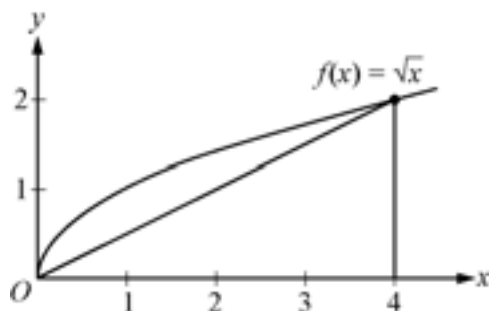
The area of the region in the first quadrant of the xy-plane below the graph of the function $f(x) = \sqrt{x}$ between $x = 0$ and $x = 4$ is estimated by using the triangle shown above. What is the difference between the estimate and the actual value of the area?

A. $\dfrac{2}{9}$

B. $\dfrac{4}{3}$

C. $\dfrac{5}{3}$

D. 4

COMPETENCY 011

20. **Use the figure below to answer the question that follows.**
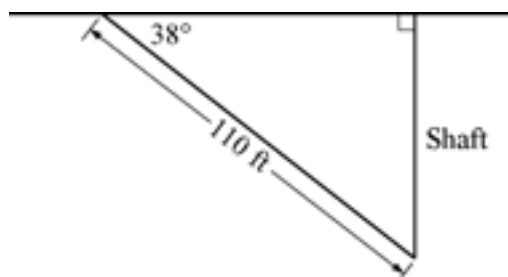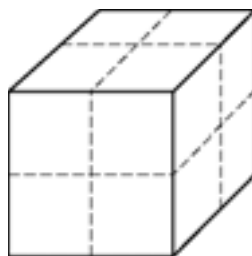

Workers at a mine have dug a tunnel that is 110 feet long and forms an angle of 38° with the ground. A vertical shaft is dug to ventilate the tunnel, as shown in the figure above. To the nearest foot, how long is the vertical shaft?

A. 68 feet

B. 86 feet

C. 87 feet

D. 98 feet

COMPETENCY 011

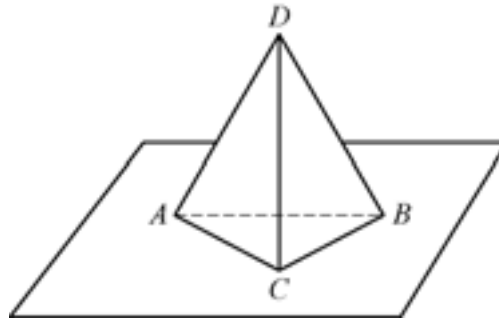21. **Use the figure below to answer the question that follows.**

The figure above represents a cube with a surface area of 216 square inches. The dotted lines bisect the faces of the cube. If each face of the cube were sliced along the dotted lines to produce smaller cubes, what would be the sum of the surface areas of all the smaller cubes produced?

A. 216 in^2

B. 432 in^2

C. 864 in^2

D. 2592 in^2

COMPETENCY 012

22. **Use the diagram below to answer the question that follows.**
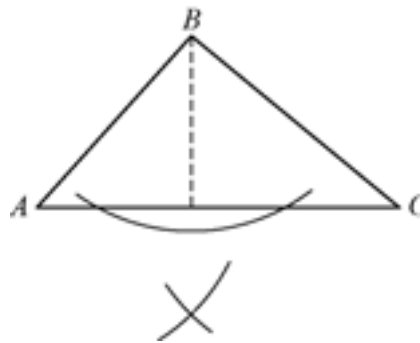

In the diagram above, $\overline{DC}$ is perpendicular to the plane of $\triangle ABC$, and $\triangle ABC$ is equilateral. Which of the following justifies the assertion that $\triangle ACD \cong \triangle BCD$?

A. AAA

B. SAS

C. SAA

D. SSS

COMPETENCY 012

23. **Use the figure below to answer the question that follows.**

The figure above shows the markings for constructing an altitude of a given triangle, *ABC*. This is based on constructing

A. a ray that bisects one of the angles of $\triangle ABC$.

B. the perpendicular bisector of one of the sides of $\triangle ABC$.

C. a point equidistant from the vertices of $\triangle ABC$.

D. a line through one of the vertices and perpendicular to one of the sides of $\triangle ABC$.

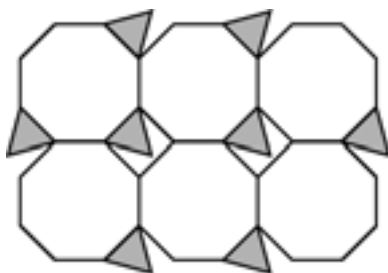24. **Use the figure below to answer the question that follows.**

A truncated cube is a polyhedron formed by cutting the corners off of a solid cube, as shown above. Which of the following nets represents a truncated cube?

A.

C.
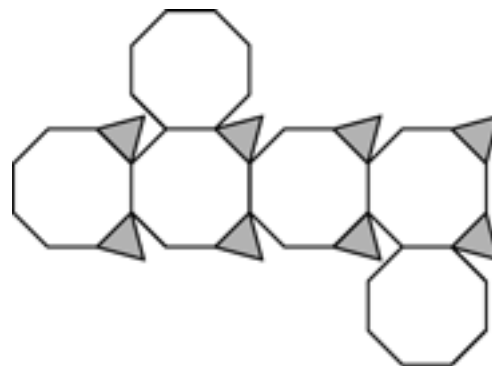
B.

D.

COMPETENCY 013

25. **Use the figure below to answer the question that follows.**

In the figure above, rectangles *ABCD* and *ECDF* are similar, $AF = AB$, and $\overline{EF}$ is perpendicular to $\overline{AD}$. If $AD = 1$ and $FD = x$, which of the following equations must be true?

A. $x^2 - x + 1 = 0$

B. $x^2 - 2x - 2 = 0$

C. $x^2 - 2x + 2 = 0$

D. $x^2 - 3x + 1 = 0$

COMPETENCY 014

26. **Use the figure below to answer the question that follows.**
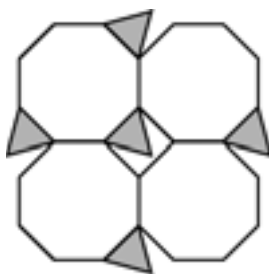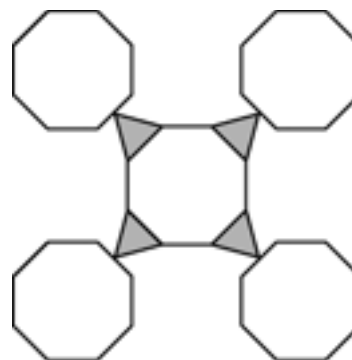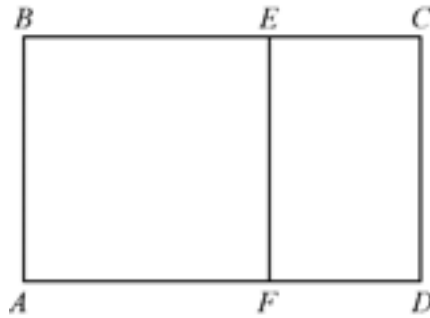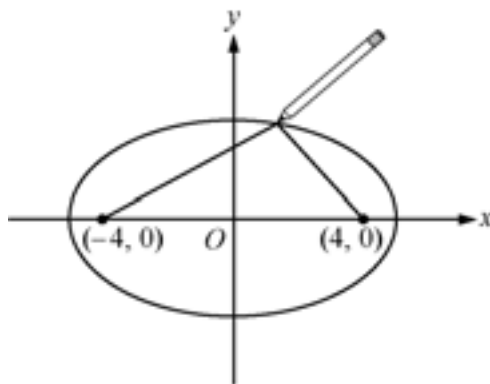

In the figure above, two thumbtacks are placed at the points $(-4, 0)$ and $(4, 0)$. The ends of a piece of string are attached to the tacks, one end to each tack, and the string is pulled tight as a pencil traces out an ellipse. If the equation of the ellipse is $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$, how many units long is the piece of string?

A. 5

B. 8

C. 10

D. 16

COMPETENCY 014

27. A search team has been hired to search for a lost hiker. The hiker was last spotted at coordinates $(28, 75)$ on the search team's map. Since being last spotted, the hiker could not have traveled more than 15 grid units. A point, (x, y), on the boundary of the search region must satisfy which of the following equations?

A. $(x + 28)^2 - (y + 75)^2 + 15^2 = 0$

B. $(x - 28)^2 + (y - 75)^2 + 15^2 = 0$

C. $(x + 28)^2 - (y + 75)^2 - 15^2 = 0$

D. $(x - 28)^2 + (y - 75)^2 - 15^2 = 0$

COMPETENCY 015

28. **Use the figure below to answer the question that follows.**
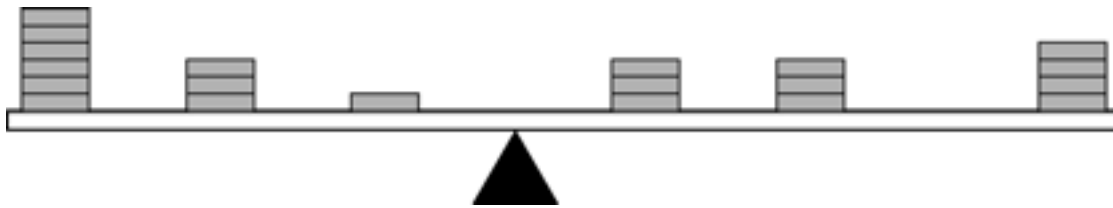
In a unit on measures of central tendency, a teacher places 20 equal weights on top of a ruler. The teacher then balances the ruler on a triangular block, as shown above. Which of the following measures of central tendency is most analogous to the point at which the ruler balances?

A. Variance

B. Mean

C. Mode

D. Median

COMPETENCY 015

29. Students in a high school class have collected data on the gas mileage of their parents' cars. The data are summarized in the table below.

Gas Mileage (miles per gallon)	
Mean	22.7
Median	24
Variance	16.8

The students would like to convert the results to kilometers per liter to compare the results with those of a class in Holland. If 1 mile per gallon ≈ 0.425 kilometer per liter, which of the following tables shows the correct conversions for each statistical measure?

A.

Gas Mileage (kilometers per liter)	
Mean	9.65
Median	24
Variance	16.8

B.

Gas Mileage (kilometers per liter)	
Mean	9.65
Median	10.20
Variance	16.8

C.

Gas Mileage (kilometers per liter)	
Mean	9.65
Median	10.20
Variance	7.14

D.

Gas Mileage (kilometers per liter)	
Mean	9.65
Median	10.20
Variance	3.03

5

COMPETENCY 016

30. **Use the information below to answer the question that follows.**

> A factory that makes air conditioners uses two assembly lines. Line 1 produces
> 6000 units per years, and 3% of the units are defective. Line 2 produces 4000 units
> per year, and 5% of the units are defective. A quality control inspector chooses
> a unit at random from among all the units produced last year and finds it to be
> defective.

Which of the following tree diagrams would be most useful to the inspector in calculating
the probability that the defective unit came from assembly line 2 ?

A.
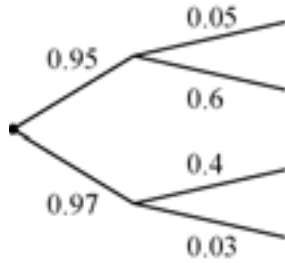
C.
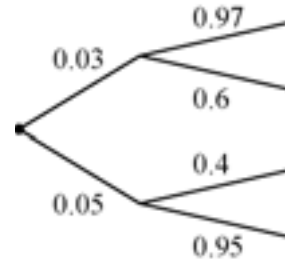
B.
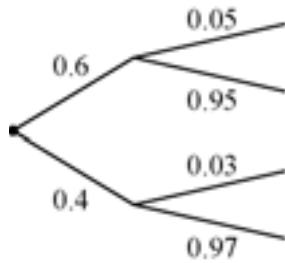
D.
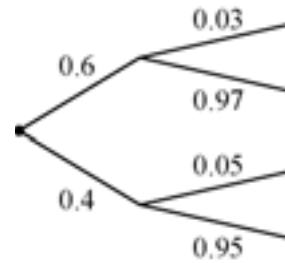
COMPETENCY 016

31. A circle is inscribed in a square. What is the probability to the nearest thousandth that a point selected at random from inside the square will also be inside the circle?

    A. 0.318

    B. 0.637

    C. 0.785

    D. 0.858

COMPETENCY 017

32. During a class demonstration, a teacher used a computer to draw 200 simple random samples of size 5 taken from the population $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and then computed the mean of each of the 200 samples. The teacher then used the computer to calculate the mean, 4.5, and the standard deviation, 2.87, of the population $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Finally, the teacher displayed a histogram of the distribution of the 200 sample means along with the graph of a normal curve with mean 4.5 and standard deviation $\frac{2.87}{\sqrt{5}}$. The main purpose of this demonstration was to illustrate

    A. the Central Limit Theorem.

    B. the Law of Large Numbers.

    C. how to find the standard z scores for the normal curve.

    D. the difference between the mean and the median of a distribution.

COMPETENCY 017

33. When a set of data (x, y) is plotted on a standard xy-coordinate system, the data points essentially lie on a nonlinear curve. When the data are transformed by taking the natural logarithm of the y-values and graphed as $(x, \ln y)$, the transformed data lie nearly on a straight line. This indicates that the initial data would be best modeled by which of the following types of functions?

    A. Exponential

    B. Quadratic

    C. Rational

    D. Trigonometric

COMPETENCY 018

34. Students are using a graphing calculator to explore how changing the value of *m* in an equation of the form *y* = *mx* changes the graph of the equation. The students view several graphs and are then asked to make a generalization about how the value of *m* affects the graph of the equation. This is an example of using

A. a counterexample to evaluate a mathematical relationship.

B. an axiomatic system to generate a mathematical relationship.

C. inductive reasoning to conjecture a mathematical relationship.

D. deductive reasoning to prove a mathematical relationship.

COMPETENCY 018

5

35. **Use the table below to answer the question that follows.**

Location	Altitude (feet)	Boiling Point of Water (°F)
Sea Level	0	212
Denver, CO	5000	203

The temperature at which water boils is a function of altitude above sea level. How well does a linear model based on the data in the table above predict the temperature (160°F) at which water boils on Mt. Everest (altitude = 29,000 feet)?

A. Very well, with an error less than 1%

B. Sufficiently well, with an error of about 4%

C. Good enough for a rough estimate, with an error of about 8%

D. Very poorly; a linear model should not be used to characterize this relationship

36. In an electric circuit, voltage is equal to the product of current and resistance. A technician measures a current of 1.2 amps when a 20-ohm resistor is connected in a circuit with a battery of unknown voltage. When the first resistor is replaced with an unlabeled resistor, the current in the circuit is 2 amps. What is the value of the unlabeled resistor?

   A. 12 ohms

   B. 33.3 ohms

   C. 48 ohms

   D. 66.6 ohms

COMPETENCY 021

37. Ms. Jones is beginning a new unit on geometry. She would like to identify aspects of the new topic that are most difficult for students to understand in order to adjust her lesson plans accordingly. Which of the following assessment methods would be most appropriate for achieving this goal?

   A. Periodic surprise quizzes

   B. Regular peer review of other students' work

   C. Performance assessments at the beginning and end of the unit

   D. Informal observations and interviews during each class

ANSWER KEY

Question Number	Correct Answer	Competency
1	A	001
2	D	001
3	C	002
4	B	002
5	C	003
6	C	003
7	C	004
8	D	004
9	C	005
10	A	005
11	A	006
12	B	006
13	D	007
14	D	007
15	D	008
16	B	008
17	C	009
18	D	009
19	B	010
20	A	011
21	B	011
22	B	012
23	D	012
24	C	013
25	D	013
26	C	014
27	D	014
28	B	015
29	D	015
30	D	016
31	C	016
32	A	017
33	A	017
34	C	018
35	A	018
36	A	019
37	D	021

Chapter 6

Are You Ready? – Last Minute Tips

★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★

PREPARING TO TAKE THE TEST

CHECKLIST

Complete this checklist to determine if you are ready to take your test.

- ✓ Do you know the testing requirements for your teaching field?

- ✓ Have you followed the test registration procedures?

- ✓ Have you reviewed the test center identification document requirements in the *Registration Bulletin* or on the ETS TExES website at **www.texes.ets.org**?

- ✓ Do you know the test frameworks that will be covered in each of the tests you plan to take?

- ✓ Have you used the study plan sheet at the end of this booklet to identify what content you already know well and what content you will need to focus on in your studying?

- ✓ Have you reviewed any textbooks, class notes and course readings that relate to the frameworks covered?

- ✓ Do you know how long the test will take and the number of questions it contains? Have you considered how you will pace your work?

- ✓ Are you familiar with the test directions and the types of questions for your test?

- ✓ Are you familiar with the recommended test-taking strategies and tips?

- ✓ Have you practiced by working through the sample test questions at a pace similar to that of an actual test?

- ✓ If constructed-response questions are part of your test, do you understand the scoring criteria for these questions?

- ✓ If you are repeating a test, have you analyzed your previous score report to determine areas where additional study and test preparation could be useful?

THE DAY OF THE TEST

You should have ended your review a day or two before the actual test date. Many clichés you may have heard about the day of the test are true. You should:

- Be well rested.

- Take the appropriate identification document(s) with you to the test center (identification requirements are listed in the *Registration Bulletin* and on the ETS TExES website at **www.texes.ets.org**).

- Take 3 or 4 well-sharpened soft-lead (No. 2 or HD) pencils with good erasers.

- Eat before you take the test.

- Be prepared to stand in line to check in or to wait while other test takers are being checked in.

- Stay calm. You can't control the testing situation, but you can control yourself. The test administrators are well trained and make every effort to provide uniform testing conditions, but don't let it bother you if a test doesn't start exactly on time. You will have the necessary amount of time once it does start. Using the *Reducing Test Anxiety* booklet in the days before you test may be helpful in mentally and emotionally preparing yourself to test. It is available free at **www.texes.ets.org**.

You can think of preparing for this test as training for an athletic event. Once you have trained, prepared and rested, give it everything you've got. Good luck.

6

Appendix A

Study Plan Sheet

A

STUDY PLAN						
Content covered on test	How well do I know the content?	What material do I have for studying this content?	What material do I need for studying this content?	Where can I find the materials I need?	Dates planned for study of content	Date completed

A

Appendix B

Preparation Resources

PREPARATION RESOURCES

The resources listed below may help you prepare for the TExES test in this field. These preparation resources have been identified by content experts in the field to provide up-to-date information that relates to the field in general. You may wish to use current issues or editions to obtain information on specific topics for study and review.

JOURNALS

American Mathematical Monthly, Mathematical Association of America.

Journal for Research in Mathematics Education, National Council of Teachers of Mathematics.

Mathematics Magazine, Mathematical Association of America.

Mathematics Teacher, National Council of Teachers of Mathematics.

OTHER RESOURCES

Bittenger, M. L., and Ellenbogen, D. (2005). *Elementary Algebra: Concepts and Applications*, Seventh Edition. Menlo Park, Calif.: Addison-Wesley.

Bock, D., Velleman, P., and De Veaux, R. (2009). *Stats: Modeling the World*, Third Edition. Boston, Mass.: Pearson Education, Inc.

Brahier, D. J. (2009). *Teaching Secondary and Middle School Mathematics*, Fourth Edition. Needham Heights, Mass.: Allyn & Bacon.

Brumbaugh, D. K., and Rock, D. (2006). *Teaching Secondary Mathematics*, Third Edition. Mahwah, N.J.: Lawrence Erlbaum Associates.

Burger, E., and Starbird, M. (2005). *The Heart of Mathematics: An Invitation to Effective Thinking*, Second Edition. Emeryville, Calif.: Key College Publishing

COMAP (2008). *For All Practical Purposes: Mathematical Literacy in Today's World*, Eighth Edition. New York, N.Y.: W. H. Freeman & Company.

Connally, E., Hughes-Hallett, D., Gleason, A., et al. (2007). *Functions Modeling Change: A Preparation for Calculus*, Third Edition. Hoboken, N.J.: John Wiley & Sons, Inc.

Cooney, T., Brown, S., Dossey, J., Schrage, G., and Wittmann, E. (1999). *Mathematics, Pedagogy, and Secondary Teacher Education*. Portsmouth, N.H.: Heinemann

Coxford, A., Usiskin, Z., and Hirschhorn, D. (1998). *The University of Chicago School of Mathematics Project: Geometry*. Glenview, Ill.: Scott, Foresman and Company.

Crouse, R. J., and Sloyer, C. W. (1987). *Mathematical Questions from the Classroom — Parts I and II*. Providence, R.I.: Janson Publications.

Danielson, C., and Marquez, E. (1998). *A Collection of Performance Tasks and Rubrics: High School Mathematics*. Larchmont, N.Y.: Eye on Education.

Demana, F., Waits, B. K., Clemens, S. R., and Foley, G. D. (1997). *Precalculus: A Graphing Approach*, Fourth Edition. Menlo Park, Calif.: Addison-Wesley.

Farlow, S. J. (1994). *Finite Mathematics and Its Applications*, Second Edition. Boston, Mass.: WCB McGraw-Hill.

Foerster, P. A. (2005). *Calculus Concepts and Applications*, Second Edition. Berkeley, Calif.: Key Curriculum Press.

Garfunkel, Godbold, and Pollak. (1999). *Mathematics: Modeling Our World. Books 1, 2 & 3.* New York, N.Y.: W. H. Freeman & Co.

Greenburg, M. (2008). *Euclidean and Non-Euclidean Geometries: Development and History*, Fourth Edition. New York, N.Y.: W. H. Freeman and Company.

Hughes-Hallett, D., Gleason, A., McCallum, W., et al. (2008). *Calculus: Single Variable*, Fifth Edition. Hoboken, N.J.: John Wiley & Sons, Inc.

Hungerford, T. W. (2004). *Contemporary College Algebra and Trigonometry: A Graphing Approach*, Second Edition. Philadelphia, Pa.: Harcourt College Publishers.

Johnson, K., Herr, T., Kysh, J. (2004). *Problem Solving Strategies: Crossing the River with Dogs*, Third Edition. Emeryville, Calif.: Key College Publishing.

Kilpatrick, J., Swafford, J., and Finell, B. (Eds.) (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, D.C.: National Academy Press.

Kinsey, L., and Moore, T. (2002) S*ymmetry, Shape, and Space: An Introduction to Mathematics Through Geometry*. Emeryville, Calif.: Key College Publishing.

Larson, R., Hostetler, R., and Edwards, B. (2005). *Calculus of a Single Variable*, Eighth Edition. Boston, Mass.: Houghton Mifflin Harcourt.

Lay, S. (2005). Analysis: *With an Introduction to Proof*, Fourth edition. Upper Saddle River, N.J.: Pearson Education, Inc.

Leitzel, James R. C. (1991). *A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics*. Washington, D.C.: Mathematical Association of America.

National Council of Teachers of Mathematics. (1995). *Assessment Standards for School Mathematics*. Reston, Va.: The National Council of Teachers of Mathematics, Inc.

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Robbins, N. (2006). *Beginning Number Theory*, Second Edition. Sudbury, Mass.: Jones and Bartlett Publishers.

B

Rosen, K. (2006). *Discrete Mathematics and Its Applications*, Sixth Edition. Boston, Mass.: McGraw-Hill Higher Education.

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Strang, G. (2005). *Linear Algebra and Its Applications*, Fourth Edition. Pacific Grove, Calif.: Brooks Cole Publishers.

Texas Education Agency. (2009). Texas Essential Knowledge and Skills (TEKS).

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B